ance circuits. For example, it is not generally appreciated that, for typical values such as $D=\lambda/4$ and $t/D=0.1$, the TE mode cut-off corresponds to a characteristic impedance of 43 ohms. Whereas the existence of this mode does not necessarily cause serious trouble, it may often explain discrepancies between experimental results and those calculated on the basis of a pure TEM mode. As is obvious from Fig. 3, this may be avoided by using a thicker strip.

The second point is the existence of an optimum characteristic impedance for obtaining the lowest attenuation. The value of this optimum will depend on the desired higher mode attenuation. Because the conditions will vary widely for different applications, the data presented cover the case of operation at the cut-off frequency of both the lowest TE and TM modes. This condition produces the lowest possible loss. For practical applications, however, the method for obtaining the optimum in other cases has been outlined.

A few final words should be said in regard to an interesting point shown by the curves in Fig. 3 and 4. It is assumed that one usually wishes to operate with the lowest possible line losses and this generally imples a high value of D, and therefore, of Df. As to the strip dimensions for lowest line loss an examination of the curves in Figs. 3 and 4 shows that for low values of characteristic impedance, it is desirable (see Fig. 4) and often necessary (see Fig. 3) to use high values of t/D. From the curves for characteristic impedance given by Bates[6] it is seen that high values of t/D imply small values of w/D. On the other hand, Bates also shows that a high characteristic impedance can *only* be obtained with small values of t/D. Therefore, it may be concluded that, in addition to the preceding considerations of the optimum characteristic impedance, one may make the generalization that a high impedance line with lowest loss should be in the familiar strip line from ($t \ll w$) whereas low impedance lines with lowest loss should be made with much thicker strips, in some cases with the strip thickness exceeding the strip width ($t>w$).

Deflection of Waveguide Subjected to Internal Pressure*

LUCIEN G. VIRGILE†

Summary—**The pressure carrying capacity of a large range of standard waveguide sizes can be readily determined by the use of formulas presented in this paper. The derivation of these formulas is obtained by a continuous beam analogy and comparable test results are shown which substantiate the validity of the theoretical analysis.**

Where high pressure conditions prevent the use of standard waveguide, these same formulas are utilized in the development of special high-strength lightweight guide. Techniques for designing such waveguide, including the use of a honeycomb sandwich construction, are discussed.

THE DEVELOPMENT of radar systems of increasing range has been brought about largely by the use of greater and greater power. In order to increase the power handling capacity of the microwave packages, pressurization is utilized to prevent electrical breakdown. It is, therefore, extremely desirable to be able to determine quickly the pressure carrying capacity of a given waveguide and, when standard waveguide cannot safely carry the required pressure, to be able to design special guide of minimum weight and/or cost.

The derivation of formulas that express the relationship of wall thickness to pressurization capacity is presented for a considerable range of waveguide sizes. The criteria for this relationship are 1) that the waveguide should not permanently distort, and 2) that the elastic deflection should not exceed the amount permissible for satisfactory microwave use. The problem is approached both analytically and empirically with good correlation between the two methods.

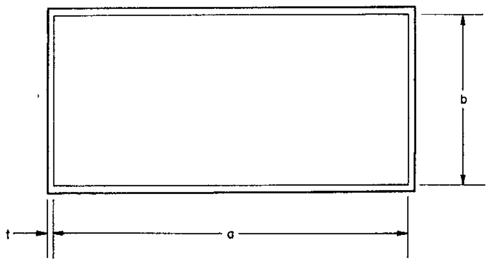
Fig. 1(a) depicts a typical cross section of unpressurized guide. The application of internal pressure results in distortion as shown in Fig. 1(b). The question frequently arises, "How can the short wall bend inward when the pressure should be forcing it outward?" A simplified explanation of this phenomenon is that the corner moment, due to the relatively greater length of the long wall, is sufficient to more than overcome the internal pressure on the short wall, resulting in an inward deflection. This is borne out by both the derived formulas and actual test results.

The pressurized waveguide cross section is considered to be similar to a uniformly loaded continuous beam of an infinite number of spans (or a simple beam with end moments[1]) as shown in Fig. 2. The analysis that follows pertains to unsupported waveguide which, as a practical consideration, means that it is applicable to sections that

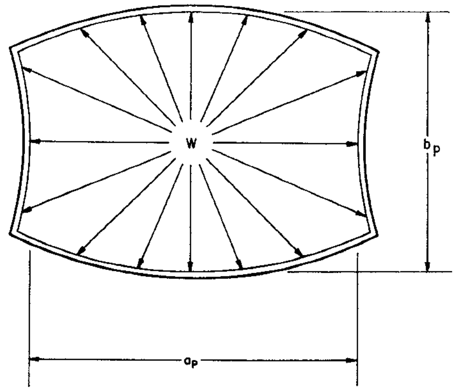
* Manuscript received by the PGMTT, March 13, 1957; revised manuscript received, May 13, 1957.

† Microwave Electronics Div., Sperry Gyroscope Co., Great Neck, N. Y.

[1] T. N. Anderson, "Rectangular and ridge waveguide," IRE Trans., vol. MTT-4, pp. 201–209; October, 1956.



(a)



(b)

Fig. 1—(a) Unpressurized waveguide; (b) waveguide under pressure.

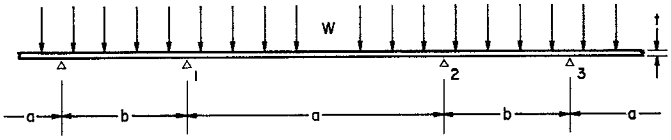


Fig. 2—Waveguide shown as infinite continuous beam.

are distant from flanges (or other supports) by a length of more than the broad wall dimension. Symbols used in the derivations are defined in Table I.

The three-moment equation is applied to the first three spans of the continuous beam resulting in:

$$\frac{M_1 a}{I_1} + 2M_2\frac{(a+b)}{I_2} + \frac{M_3 b}{I_3} = \frac{w(a^3+b^3)}{4I}\,.$$

The beam (for standard waveguide) is of constant cross section, allowing the I term to drop out. Since the waveguide is symmetrical at each corner,

$$M_1 = M_2 = M_3 = M \text{ and}$$

$$3M(a+b) = \frac{w}{4}(a^3+b^3)$$

$$M = \frac{w(a^3+b^3)}{12(a+b)}\,. \tag{1}$$

It may be noted that (1) is consistent with the common structural practice of considering the maximum bending moment on any span of a continuous beam of many equal spans to be equal to $wL^2/12$.

TABLE I

SYMBOLS AND ELEMENTARY EQUATIONS

a = inside long dimension in inches
b = inside short dimension in inches
t = thickness in inches
w = pressure in pounds per square inch*
$I = t^3/12$ = moment of inertia*
M = bending moment at any corner or point of support (since $M_1 = M_2 = M_3 = M_4$, etc.)
E = modulus of elasticity in pounds per square inch
s = stress at yield point** in pounds per square inch
k_1, k_2, k_3 = constants for given values of a, b, and t
y_a = deflection of a at $a/2$ in inches
y_b = deflection of b at $b/2$ in inches
$a_p = a + 2y_b$ = dimension a when presssurized
$b_p = b + 2y_a$ = dimension b when pressurized
$c = t/2$ = distance to most remote fiber
X = an unknown arbitrary length
L = a known arbitrary length

* Based on unit length to permit simplified analysis.
** The limit to which the analysis is valid since beyond this point strain and stress are no longer proportional.

Using the basic relationship between stress and strain, and substituting (1), the formula for pressure becomes:

$$S = \frac{Mc}{I} = \frac{\dfrac{w(a^3+b^3)}{12(a+b)}\,\dfrac{t}{2}}{\dfrac{t^3}{12}} = \frac{w(a^3+b^3)}{2t^2(a+b)}$$

$$w = \frac{2st^2(a+b)}{(a^3+b^3)}\,. \tag{2}$$

Thus, by substituting the known dimensions a, b, and t for a given size waveguide and the allowable stress s for a given material, the maximum allowable pressure is easily obtained from (2). Conversely, the same formula can be utilized to solve for s or t for a given pressure.

To determine the waveguide deflection under pressure, it is necessary to combine the deflections due to load and moment.[2] Deflection due to load is expressed as

$$y = \frac{5}{384}\frac{wL^4}{EI},$$

and deflection due to moment as

$$y = 2\left\{\frac{1}{6}\frac{M}{EI}\left(3x^2 - \frac{x^3}{L} - 2Lx\right)\right\}\,.$$

The direction of the moment is such that it opposes the effect of the load. Thus,

$$y_a = \frac{5}{384}\frac{wa^4}{EI} - \frac{Ma^2}{8EI}$$

$$y_a = \frac{5}{384}\frac{wa^4}{EI} - \frac{wa^2(a^3+b^3)}{96EI(a+b)}$$

$$y_a = \frac{w}{E}\left\{\frac{5}{32}\frac{a^4}{t^3} - \frac{a^2(a^3+b^3)}{8(a+b)t^3}\right\} \tag{3}$$

[2] R. J. Roark, "Formulas for Stress and Strain," McGraw-Hill Book Co., Inc., New York, N. Y., p. 109; 1954.

TABLE II
WAVEGUIDE CONSTANTS

a	b	t	k_1	k_2	k_3	w*
6.500	3.250	0.080	0.000404	215,000	−48,000	2.0
4.300	2.150	0.080	0.000920	43,400	−9,200	4.6
3.400	1.700	0.080	0.00148	16,400	−3,570	7.4
2.840	1.340	0.080	0.00211	7,880	−1,650	10.6
1.872	0.872	0.064	0.00309	2,940	−619	15.5
1.372	0.622	0.064	0.00579	836	−171	29
1.122	0.497	0.064	0.00864	379	−76	43
0.900	0.400	0.050	0.00820	585	−66	41
0.622	0.311	0.040	0.01103	149	−32	55
0.420	0.170	0.040	0.02388	30	−5	119

* Maximum pressure (in pounds per square inch gauge) that may be utilized without permanent deformation in annealed standard aluminum waveguide.

TABLE III
S AND E VALUES

Waveguide Material	S—As received	S Annealed	E
2S Aluminum	13,000 psi	5000 psi	10^6 psi
61S Aluminum	35,000 psi		10^6 psi
Brass	40,000 psi	12,000 psi	1.2×10^6 psi

and similarly,

$$y_b = \frac{w}{E}\left\{\frac{5}{32}\frac{b^4}{t^3} - \frac{b^2(a^3+b^3)}{8(a+b)t^3}\right\}. \tag{4}$$

Thus, the deflections are expressed in simple known terms. These deflections are then added to the original inside measurements to find the inside dimensions under pressure as follows:

$$a_p = a + 2y_b$$

$$b_p = b + 2y_a.$$

Eqs. (2)–(4) can also be expressed as

$$w = k_1 s \tag{5}$$

$$y_a = k_2 \frac{w}{E} \tag{6}$$

$$y_b = k_3 \frac{w}{E} \tag{7}$$

where k_1, k_2, and k_3, are functions of a, b, and t. To avoid future repetitious calculations, Table II has been prepared showing these constants for various sizes of standard waveguide.

Values of E and s are required to complete the solutions and may be obtained from a number of standard reference sources. A caution to be observed lies in the selection of suitable s values, since the waveguide material may be annealed in the making of the microwave component (for example, in attaching flanges by oven brazing). Table III shows values for E and s that have proved to be satisfactory.

A substantial number of tests have been performed at the Sperry Gyroscope Company that show close agreement between experimental results and values calculated from the above formulas. Typical comparisons are shown in Fig. 3 and Fig. 4. These curves represent a specimen of 2S aluminum L-band waveguide with heli-arc welded (no annealing) flanges. Tests on other sizes of guide have yielded similarly good correlation to the theory. The slight discrepancies that exist between calculated and test values are probably due to variations in waveguide dimensions (calculated values are based on nominal waveguide size) and/or inhomogeneous modulus of elasticity as well as test inaccuracies.

It should be borne in mind that it is generally unsatisfactory to determine pressure carrying capacity solely from the stress point of view. In most cases it is necessary also to calculate the deflection with consideration of the specific application. For example, the waveguide

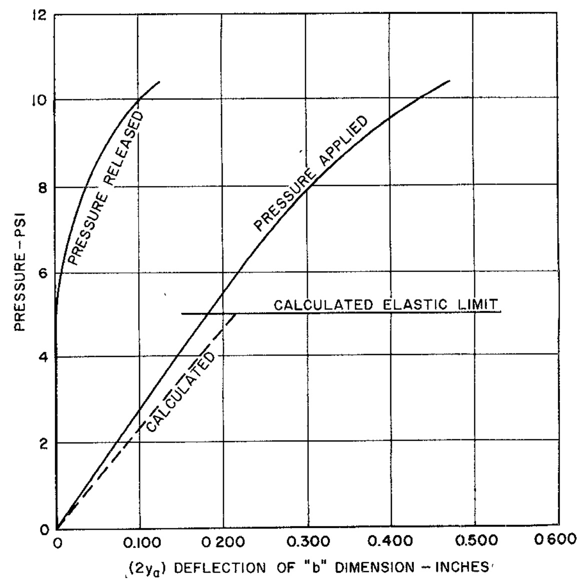


Fig. 3—Effect of pressurization on 2S aluminum L-band waveguide (WR 650).

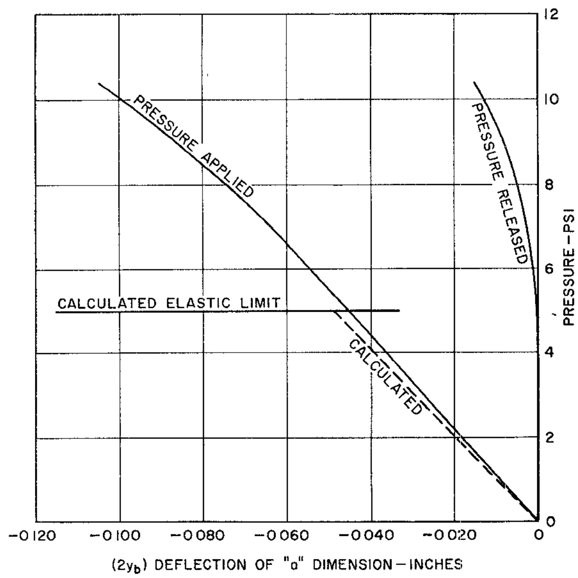


Fig. 4—Effect of pressurization on 2S aluminum L-band waveguide (WR 650).

deflection in an impedance transformer would be limited to a few thousandths of an inch to prevent power leakage past the shorts and the stress in the guide walls would, consequently, be very small. On the other hand, a piece of interconnecting waveguide might be permitted to deflect to 5 per cent or more over its original dimensions, in which case the yield stress could well become the limiting factor. In passing, it should be noted that it is seldom necessary to calculate the change in the broad dimension as this will be less than one-quarter of the narrow dimension change.

Where pressures are involved that are in excess of the capacity of the standard waveguide under question, a number of strengthening procedures can be employed. Most of the reinforcement methods can be expressed as an increase of the average wall thickness or section modulus, thus making possible the use of (1)–(4) to determine the required increase in section. Simplified formulas (5)–(7) can also be employed for many such cases by recalling that pressure-carrying capacity varies directly as the square of wall thickness and that deflection varies inversely as the cube of wall thickness.

Perhaps the most common means of strengthening waveguide is the addition of brazed, welded, or bolted braces, the design of which at minimum weight and cost involves considerable ingenuity. A second method is that of simply increasing the over-all waveguide wall thickness, utilizing such processes as precision cored sand casting or by welding plate together to form a waveguide; under such conditions, a useful but generally overlooked weight advantage can be obtained by making the narrow wall thinner than the broad wall, since, as noted in a previous paragraph, the change of the broad dimension under pressure is less than one-quarter that of the narrow dimension change. It is also possible to construct waveguide that is both stronger and lighter than standard guide. Successful means of accomplishing this end are 1) reinforced plastic guide coated with conductive metallic resin, and 2) a waveguide wall construction composed of two thin sheets of aluminum sandwiched over an aluminum honeycomb core. This latter method has resulted in construction of L-band honeycomb waveguide weighing 60 per cent of standard aluminum guide and increasing strength by a factor of four; it also appears likely that if the weight of the honeycomb guide is made equal to standard waveguide, the strength ratio would be approximately twenty to one. Both the reinforced plastic and honeycomb waveguide have been built and the former has already been utilized in radar systems by the Microwave Electronics Division of the Sperry Gyroscope Company.

It is apparent that the wall thicknesses of standard waveguide were not established with regard to pressure carrying capacity. As shown in the last column of Table II, a wide variation exists in this respect for the different waveguide sizes. Specially drawn material is available[1] to carry high pressure in large size guide. It would seem worthwhile to carry this approach a step further and provide reduced wall thicknesses in the smaller sizes to obtain lighter weight, particularly for airborne applications wherein pressures are not too great. With reasoning of this type as a basis, it appears that an investigation of a revised standardization for waveguide wall thicknesses would be justified.

The Calibration of Microwave Attenuators by an Absolute Method*

ELIZABETH LAVERICK†

Summary—**A bridge method by means of which microwave attenuators can be calibrated absolutely is described, with a consideration of the main possible sources of error. A bridge was set up at $\lambda_0 = 3.2$ cm to test the principle of the method. It was shown that, using nonspecialized equipment, a high degree of accuracy was obtainable. An attenuator was calibrated over a range of 20 db, with an accuracy of the order of ± 0.02 db. This accuracy is within the accuracy of other methods of calibration in current use, and there seems no reason why, with suitable precautions, the order of accuracy should not be improved still further, if required.**

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Introduction

THE MAIN method in current use of calibrating microwave attenuators involves calibration in terms of a standard attenuator, usually the piston attenuator,[1–3] whose law of attenuation is accurately

[1] C. G. Montgomery (ed.), "Technique of Microwave Measurements," M.I.T. Rad. Lab. Ser. No. 11, McGraw-Hill Book Co., Inc., New York, N. Y., Ch. 11 and 13; 1947.

[2] L. G. H. Huxley, "A Survey of the Principles and Practice of Waveguides," Cambridge University Press, Cambridge, Eng., pp. 57–61; 1947.

[3] G. F. Gainsborough, "A method of calibrating standard signal generators and radio-frequency attenuators," *J. IEE*, pt. III, vol. 94, pp. 203–210; May, 1947.